INDIAN SCHOOL AL WADI AL KABIR

Assessment - I (2023-24)

SUB: Applied Mathematics (241)

Date: 26/09/2023   Set I   Time Allowed :3 hours

Grade: XII   Maximum Marks: 80

General Instructions:

1. This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
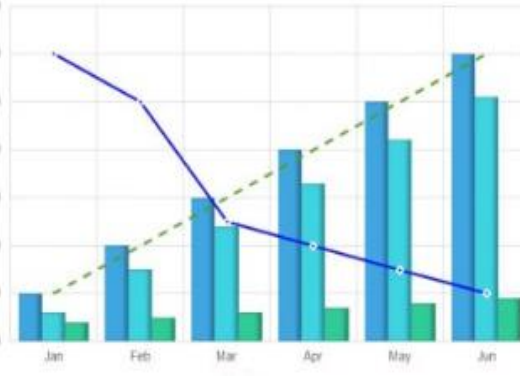
Q. No	SECTION A (MCQ)								Marks
1.	If a matrix has 16 elements, then which of the following cannot be its order?								1
	A	2×8	**B**	4×4	**C**	1×16	**D**	15×1	
2.	$\begin{vmatrix} 2x & 4 \\ -3 & x \end{vmatrix} = \begin{vmatrix} 5 & 0 \\ -1 & 6 \end{vmatrix}$ then value of x.								1
	A	± 3	**B**	$\pm 2\sqrt{6}$	**C**	± 5	**D**	$\pm 2\sqrt{7}$	
3.	$If\ A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ -1 & 0 & 3 \end{pmatrix}, |adjA| = $ ___								1
	A	3	**B**	9	**C**	27	**D**	81	
4.	*Write the sum of order and degree of differntial equation* $\left(\dfrac{dy}{dx}\right)^{\frac{1}{2}} = \left(\dfrac{d^2y}{dx^2}\right)^{\frac{1}{3}}$								1
	A	5	**B**	4	**C**	3	**D**	Not defined	

5.	If $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ then A^{-1}	1

A	$\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$	**B**	$\begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix}$	**C**	$\begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{bmatrix}$	**D**	$\begin{bmatrix} 2 & \frac{5}{2} \\ 1 & \frac{3}{2} \end{bmatrix}$

6.	If the mean and the variance of a binomial distribution are 4 and 2 respectively, then the probability of two successes is	1

A	$\frac{1}{2}$	**B**	$\frac{7}{64}$	**C**	$\frac{219}{256}$	**D**	$\frac{37}{56}$

7.	The value of x if A is a singular matrix, where $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -3 \\ x & 3 & 0 \end{pmatrix}$	1

A	3	**B**	2	**C**	1	**D**	0

8.	If x is a Poisson variable and $P(x = 1) = 2p(x = 2)$, then $P(x = 0) =$ _	1

A	e^{-1}	**B**	e	**C**	1	**D**	e^2

9.	If $x = \log t$ and $y = \frac{1}{t}$ then $\frac{dy}{dx} =$ _____	1

A	$-t^3$	**B**	$-\frac{1}{t}$	**C**	$-t$	**D**	e^t

10.	Given that mean of a normal variable X is 12 and standard deviation is 4. Then Z-score of data point 20 is equal to _____	1

A	1	**B**	2	**C**	3	**D**	4

11.	Which of the following is the solution of $xdy = ydx$?	1

A	$x - y = c$	**B**	$xy = c$	**C**	$\frac{y}{x} = c$	**D**	$x + y = c$

12.	The differential equation of family of parabolas whose equation is $y^2 = 4ax$ is _____	1

A	$x\frac{dy}{dx} = 2y$	**B**	$\frac{dy}{dx} = y$	**C**	$y\frac{dy}{dx} = 2x$	**D**	$x\frac{dy}{dx} = 2y^2$

13.	In which of the following interval $y = x^3 - 3x$ is strictly decreasing?	1

A	$(-\infty, -1)$	**B**	(-1, 1)	**C**	$(1, \infty)$	**D**	$(-1, \infty)$

14.	$\int_{-1}^{1} \log\left(\frac{2+x}{2-x}\right) dx = $ ___.								1
	A	0	**B**	1	**C**	2	**D**	3	

15.	A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?								1
	A	80π	**B**	400π	**C**	100π	**D**	40π	

16.	$f(x) = x^x$ has a stationary point at								1
	A	x= e	B	x=1	C	$x = \dfrac{1}{e}$	D	$x = 0$	

17.	$\int 3^{3^x} 3^x dx = $ ____.								1
	A	$\dfrac{3^{3^x}}{(\log 3)^2}$+C	**B**	$3^{3^x} + C$	**C**	$3^{3^x} \log 3 + c$	**D**	$3^{3^x} 3^x + C$	

18.	$\int_{0}^{1} e^x \left(\dfrac{x}{(x+1)^2}\right) dx$								1
	A	e^2	**B**	$\dfrac{e}{2}$	**C**	1	**D**	$\dfrac{e-2}{2}$	

ASSERTION-REASON BASED QUESTIONS

In the following questions (19 and 20), a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

A) Both A and R are true and R is the correct explanation of A.

B) Both A and R are true but R is not the correct explanation of A.

C) A is true but R is false.

D) A is false but R is true.

19.	(A) If $\begin{bmatrix} 5 & y \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3x - y & x - y \\ x & 2x - 3 \end{bmatrix}$, then $x = 2$ and $y = 1$ (R) Two square matrices A and B are equal if $|A| = |B|$.			1
	A	B	C	D

20.	(A) $f(x) = e^{2x}$ is increasing in R, where R is the set of real numbers. (R) f(x) is increasing if $f'(x) \geq 0, x \in R$, where R is the set of real numbers.	1

	A	B	C	D	

<div align="center">SECTION B</div>

21.	The probability distribution of X is given by	2

X	0	1	2	3	4
P(X)	0.1	k	2k	2k	k

Find the value ok and hence evaluate $P(x < 2)$.

22.	Find the equation of tangent to a curve y $=x^3 + 3x + 1$ at (1, 5)	2

<div align="center">OR</div>

The cost function C(x) of a commodity is given by $c(x) = \frac{2x^2+3x}{x+2}$.
Prove that marginal cost increases as the output x increases.

23.	Evaluate: $\int_3^9 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{12-x}} dx$	2

<div align="center">OR</div>

The marginal revenue of a company is given by MR = 80+20x+3x^2,
where x is the number of units sold for a period. Find the total revenue function R(x)
if at x=2, R(x) = 240

24.	Express $A = \begin{pmatrix} 3 & 4 & 0 \\ -1 & 2 & 3 \\ 1 & 2 & -1 \end{pmatrix}$ as a sum of symmetric and a skew symmetric matrix.	2

25.	If A $= \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 3 \end{bmatrix}$, and $B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ -1 & 0 & 4 \end{pmatrix}$ then find the product AB	2

<div align="center">SECTION C</div>

26.	A fair coin is tossed 9 times. Find the probability of getting
	   i) exactly 5tails;
	   ii) at least 5tails;
	   iii) at most 5 tails.
		3

27.	Find the matrix X , if $X \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$	3

28.	Solve the following equations using Cramer's rule: $3x + 4y = 24;\ 4x - 3y = 7$	3

29.	Find the consumers' surplus for the demand function $p = 25 - x - x^2$ when $p_0 = 19$. OR	3
	Find the producers' surplus for the demand function $p = 4 - 5x + x^2$ when $p_0 = 18$.	
30.	Evaluate: $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1} dx$ **OR** Evaluate: $\int_{0}^{3} |x-1| + |x-2| dx$	3
31.	Find the particular solution of the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$ when $x = 1$ and $y = -1$	3

<div align="center">SECTION D</div>

32.	In a district, exam scores of 500 student of class XII are recorded at the end of the session.
	    a) Tom scored 640 marks in total out of 800. The average score for the batch was 520 and the standard deviation was calculated to be 80. Find out how has Tom scored compared to his batch mates in the whole district.
	    b) Jerry scored 400 marks in the same batch. What can you say about her performance as compared to the batch of 500 students?
	    c) How much has Hari scored if he has done better than 45.22% of his batchmates?
	 (Given: $P(z < 1.5) = 0.9332,\ P(z < -0.12) = 0.4522$)
		5

33.	If $x^m y^n = (x+y)^{m+n}$ then prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$ **OR**	5

	A firm has the following total cost and demand functions:
	$C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$
	    (i) Find the total revenue function in terms of x.
	    (ii) Find the total profit function P in terms of x.
	    (iii) Find the profit maximum level of output of x.
	    What is the maximum profit, taking rupee as a unit of money?	

34.	Solve the following equations using matrices: $x + y + z = 35,\quad 2x - y + z = 35,\quad x - z = 15$	5
35.	Evaluate: $\int \frac{2x-3}{x^3-x^2-x+1} dx$ OR $\int \frac{x^2}{x^2-2x-3} dx$	5

<div align="center">SECTION-E</div>

36.	Ms. Rajni deposited Rs.10,000 in a bank that pays 4% interest compounded continuously.
	Based on the above information
	    i) formulate a differential equation and find its particular solution.
	    ii) How much amount will she get after 10 years?
	    iii) How long it will take to double the amount?
	(Given: $e^{0.4} = 1.4918$ and $log2 = 0.6931$)
		4

37.	Profit function of a company is given by $p(x) = 41 + 72x - 18x^2$
	
	    i) Find the profit when x= 1.
	    ii) In which interval p(x) is strictly increasing?
	    iii) (a) Find the maximum profit?
	         OR
	    iii) (b)Find the absolute minimum value of p(x) in [0, 3]
			4

38.	A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs.
	    Based on the above information, answer the following questions:
	i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use: $e^{-6} = 0{\cdot}0024$)
	ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs.
	iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb.
	         OR
	(iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs.
			4

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